M. Sc. (MATHS.)

Paper IX CC—09

TOPOLOGY

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T_0 - Space / or Kolmogorov space: Let (X, T) be a topological space. Let G be a T-open set.

Let $x, y \in X$ be two points such that $x \neq y$, i.e. x and y are distinct.

Then (X,T) is called a T_0-space $\boxed{\text{iff}}$

$\exists$ a T-open set G such that

$x \in G$ and $y \notin G$ $\boxed{\text{OR}}$

$y \in G$ and $x \notin G$.

i.e. for distinct pair of points x, and y of X there exists a T-open set G such that it contains either of x and y and not the other.

This is also known as T_0-~~space~~ axiom.

<u>Examples</u> <u>1.</u> Every discrete space is a T_0-space, because $\exists$ a T-open set $\{x\}$ which contains x but not y, where $x \neq y$.

<u>2.</u> Indiscrete topology is not a T_0-space.

Reason: only T-open sets are X, ϕ

Now X <u>contains x but also y</u>.

There is no T-open set which contains ~~either~~ one point but not the other.

<u>3.</u> A stronger T-space than a T_0-space is also a T_0-space.

<u>Reason</u> Let (X, T_1) is a T_0-space.

$\Rightarrow \exists$ a T_1-open set G s.t. $x \in G$ but $y \notin G$.

~~but~~ T_2 is stronger than T_1

$\Rightarrow G$ is a T_1-open $\Rightarrow G$ is also a T_2-open (because T_2 is stronger)

s.t. $x \in G$ and $y \notin G$

$\Rightarrow (X, T_2)$ is a T_0-space.